A new approach to predicting web wrinkling

Dilwyn P Jones[1]

[1]Emral Ltd, Yarm, UK

ABSTRACT

This paper describes a new approach to understanding and predicting wrinkles in webs in process machinery. It is based on Tension Field Theory, where the web in a span containing troughs is replaced by a planar membrane under stress in the trough direction only. The trough pattern, and hence the stress field, are uniform in the central portion of wide webs between misaligned rollers, and the theory for this case is presented. The analysis gives new insights on the importance of cross-direction strain from the preceding span, and on the gradual development of trough amplitude, which may lie below a threshold of visibility.

Conditions for the existence of troughs and wrinkles are derived and tested against published data. However, the incoming cross-direction strain and the visibility threshold for troughs are not available, so comparison is not possible. Critical conditions for wrinkling on a roller agree well in one case: unfortunately, the other data are for webs that are too narrow for the current analysis to be accurate.

The new theory predicts that web leaves the span with a shear strain. This can add to misalignment effects downstream, thus increasing the likelihood of wrinkling.

In the future, the tension field could be extended to narrower webs, roller diameter profiles and other cases by allowing trough direction to vary across the width.

Keywords: elastic buckling, misalignment, model, roller, shear, tension, tension field, trough, web, wrinkled membrane, wrinkle.

NOMENCLATURE

CD	cross direction
E	Young's modulus of web
h	web thickness
L	span length
MD	machine direction
R	roller radius
T	web tension per unit width
W	web width
x	machine direction component
y	cross direction component
α	angle between troughs and MD, stress or strain component in that direction
β	perpendicular direction to α
¥	shear strain component
£	strain component
$£_b$	critical strain for buckling into troughs
$£_t$	excess width as a fraction
$£_0$	incoming tension expressed as strain
$£_1$	CD strain, less Poisson contraction
¬	Poisson's ratio of web
σ	direct stress component
σ_b	critical stress for buckling into troughs
σ_c	critical stress for buckling into wrinkles
σ_0	incoming stress
θ	in-plane roller misalignment

1. INTRODUCTION

Thin web products are susceptible to damage when narrow wrinkles (also termed creases and folds) form as the web passes around rollers during manufacture and converting. An extensive experimental and theoretical investigation by Good et al. at the Web Handling Research Center at Oklahoma State University [1] has led to predictive models, based on elastic buckling theories. The web is planar and under uniform uniaxial tension between perfectly cylindrical, aligned rollers; but when misalignment or diameter variation increases, in-plane bending occurs, producing shear stress. This combines with the tension to give a compressive minor principal stress, usually close to the cross direction (CD). Only a small value is needed to exceed the critical value for buckling in a web span, and the web forms *troughs*, a sinusoidal out-of-plane displacement. At a large misalignment or diameter taper, a second critical point is reached when one or more of the troughs changes into a narrow wrinkle on the downstream roller, termed Regime 1 behavior. Earlier work [2], and that of Hashimoto [3], used an oversimplified bending model without tension, and found that wrinkles were seen where their incorrectly calculated trough threshold was reached.

At low tension, it becomes more difficult to form the narrow wrinkles, and Shelton [4] explained this Regime 2 behavior as the result of a rise in the compressive stress from zero at the web edges on the downstream roller, with a gradient determined by friction from the tension pressure. For a wrinkle to form, the web width must exceed the value for which the critical stress to buckle is reached at the centerline. The critical value is well predicted as the axial load to buckle a cylindrical shell [5], even though the effects of tension and contact with the rigid roller are ignored [6]. The effective friction (termed traction) falls as linespeed increases because of air entrainment, so higher tension is needed to avoid this Regime 2 behavior.

The planar web bending model predicts trough occurrence but falls short of predicting the larger compressive stresses needed for wrinkles on the roller. Instead, Good et al. [1] used Finite Element Analysis software for post-buckling analysis, and showed that the required compressive stress could be reached. More recent simulations account for the sinusoidal deformation in the troughs [7], but the early work [1] used *wrinkled membrane* elements, which replace the actual material in the troughs by one which remains planar, but has modified properties in areas where the critical stress to form troughs would otherwise be exceeded.

When wrinkled, the membrane elements use *Tension Field Theory* to calculate stress and strain. This was developed over 80 years ago [8] for beam structures containing flat, thin panels that buckle, but still contribute to strength. Applied to a web, stress is carried in the trough direction only, with the perpendicular and shear components equal to zero. Generally, the troughs can be parallel, converge or diverge, and their orientation is determined by loads and displacements on the boundaries.

This paper considers the use of Tension Field Theory to predict troughs and wrinkles in web handling. Instead of considering the bending of a planar web, the analysis starts from a web containing troughs, with angles and stresses determined by the entry and exit conditions of a single span. Although the stress perpendicular to a trough is zero, there is a compressive strain component in that direction. If it is larger than the Poisson contraction caused by the stress along the trough, then there is extra length, which has no consequence according to the modified material properties being used. However, in reality, the extra length is accommodated in the sinusoidal shape of the troughs. On the other hand, if the perpendicular strain is smaller than the Poisson contraction, there is no extra length to form a trough and so the web remains flat with its original material properties. When the web wraps the roller, the trough may generate a wrinkle with similar orientation. Alternatively, the trough will flatten if that generates less compressive stress than the critical value for wrinkling.

Wide webs between misaligned rollers form regularly spaced, parallel troughs over a large portion of the width. This suggests a simple analysis, which should be valid for large width-to-span ratios (*W/L*), and may give additional insight for all geometries. Previous work has focused on *W/L* < 1, whereas plastic film and paper are made 10 m wide with spans commonly less than 2 m. Papers by Walker et al [9, 10] highlighted the need for more work analyzing and testing wide webs, and raised the possibility that the theory for narrow webs may not be applicable.

2. PROBLEM DEFINITION

The steady state transport of a web between rollers misaligned by a small angle ¯ radians in the web plane results in parallel troughs at a potentially large angle ~ to the machine direction (MD), as shown in Figure 1. They point towards the shorter edge downstream, but move towards the long edge as the web is transported. If wrinkles form on the downstream roller, they too move sideways. The movement is expected to follow the *belt tracking equation* [11], but this has not been accurately checked. The troughs are waves which move through the web material, much like waves on water.

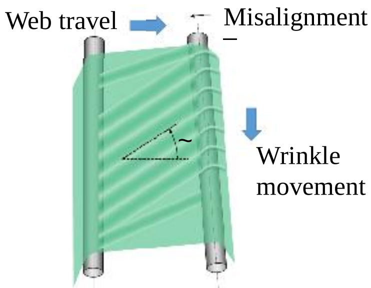

Figure 1: Troughs at an angle ~ in the span between rollers misaligned by angle ¯, causing wrinkles moving laterally.

In order to analyze the trough pattern, the web is assumed to be isotropic, linear elastic and free from camber or other defects. The friction is taken as sufficient to ensure the web enters the downstream roller matching the direction and speed of surface motion (Normal Entry, Normal Strain), and there is a zone free from slip on both upstream and downstream rollers. There are transition zones between these slip-free zones just before and after the line of contact, assumed short in the MD. In the entry transition zone, the web adjusts its strain and the trough amplitude increase from zero, both reaching the values in the main part of the span. In the exit transition zone, again before and after the contact line, the troughs collapse and compressive stress perpendicular to the troughs increases. In both zones, the web width does not change. For the analysis, the zones are assumed to be negligibly short in the MD. Approaching the edges of the span, the trough angle changes gradually to zero over a distance assumed to be small compared with the full width. This is not modelled here. The web in the previous span is assumed to be carrying uniform MD tension, and lateral contraction given by Poisson's Ratio.

When tension field theory is applied, the troughed web is replaced by a planar web with modified properties (Figure 2). Using suffix ~ to denote the trough direction, the stress and strain are related through Young's Modulus *E*:

$$\sigma_\alpha = E\varepsilon_\alpha \tag{1}$$

The stress perpendicular to the trough is taken to be zero:

$$\sigma_\beta = 0 \tag{2}$$

Shear stress and strain in $\alpha\beta$ coordinates are both zero. The strain perpendicular to the troughs ε_β is non-zero, and determined by the boundary conditions as in the next section.

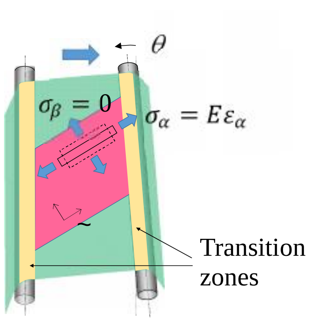

Figure 2: Troughs replaced by modified material (pink) under stress. The flat, stress-free dashed rectangle deforms into the solid rectangle under the tension field.

These stresses and strains can be transformed using standard equations to coordinates *x*,*y* based on the upstream roller. Figure 3 shows the deformed shape of the web. It is stretched by the applied tension, shows lateral contraction and is also sheared towards the short edge. All components of stress and strain are constant between the two transition zones, and away from the web edges.

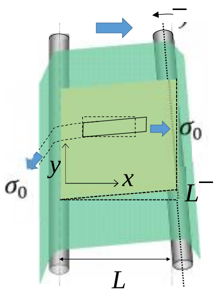

Figure 3: Deformation in *x*,*y* coordinates showing the deformation of the dashed rectangle into the parallelogram with solid outline, and the lateral displacement in the span.

3. TROUGHS

The MD tension per unit width T in the span is given, and can be converted to a stress σ_0. Transforming to $\alpha\beta$ gives:

$$\sigma_0 = T/h = E\varepsilon_0 = \sigma_\alpha \cos^2\alpha \quad (3)$$

Transforming strain gives:

$$\varepsilon_y = \varepsilon_\alpha \sin^2\alpha + \varepsilon_\beta \cos^2\alpha \quad (4)$$

In addition to Poisson strain $-\nu\varepsilon_0$, the CD strain may have a contribution ε_1 from the extra Poisson contraction that would occur because of the tension change leaving the upstream roller. Spreading or gathering effects could be included here also.

$$\varepsilon_y = -\nu\varepsilon_0 + \varepsilon_1 \quad (5)$$

Denoting $\tan\alpha$ by t and using trigonometric identities results in:

$$\varepsilon_\beta = \left(\varepsilon_1 - \varepsilon_0(\nu + t^2)\right)(1 + t^2) \quad (6)$$

The *x*-displacement is independent of *y*, so the shear strain γ_{xy} is equal to the derivative of *y*-displacement with respect to *x*. Assuming no change through the downstream transition zone, and the web velocity direction matches that of the roller surface at the contact line (Normal Entry), then the shear strain is equal to the roller angle θ [11]. Again using the transformation of strain:

$$\theta = \gamma_{xy} = -(\varepsilon_\beta - \varepsilon_\alpha)\sin 2\alpha \quad (7)$$

$$\theta = 2t(\varepsilon_0(1 + \nu + t^2) - \varepsilon_1) \quad (8)$$

For troughs to exist, the strain ε_β must be more negative than the Poisson contraction $-\nu\varepsilon_\alpha$ caused by the stress along the troughs. The extra length, denoted by ε_t, is available first to reach the buckling stress, and then to produce troughs with a non-zero amplitude. It is given by:

$$\varepsilon_t = -(\varepsilon_\beta + \nu\varepsilon_\alpha) = (\varepsilon_0 t^2 - \varepsilon_1)(1 + t^2) \quad (9)$$

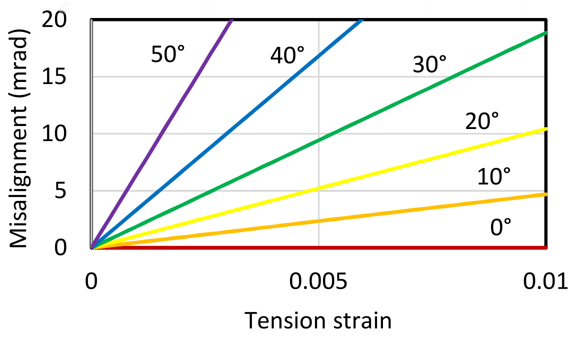

Figure 4: Contours of constant trough angle α

Contour plots of the trough angle α and the extra length ε_t as functions of tension strain ε_0 and misalignment angle θ are shown in Figures 4 and 5 respectively, for ε_1=0. The contours of α are straight lines through the origin, becoming more widely spaced as α increases. The contours of ε_t have the form $\theta \sim \sqrt{\varepsilon_0}$, similar to previous theories on wrinkling.

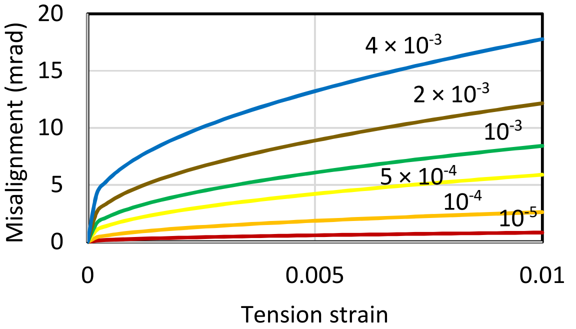

Figure 5: Contours of constant extra length £$_t$.

Presented in this way, only tension strain, extra CD strain, misalignment angle and Poisson's ratio have an influence. There is no dependence on span length, width, thickness or modulus. However, narrow spans cannot accommodate large values of ~, because troughs cannot cross the web edges. For the web centerline to lie entirely within a region of constant ~ requires:

$$\alpha < \alpha_{\max} = \tan^{-1}(W/2L) \qquad (10)$$

$$\theta < (W/L)(\varepsilon_0(1 + \nu + (W/2L)^2) - \varepsilon_1) \qquad (11)$$

The accuracy of the wide web theory is expected to be worse as θ is increased towards the value of the right hand side of equation 11. A variation in ~ across the width is needed in the analysis, but is beyond the scope of this paper.

A positive extra CD strain £$_1$ inhibits trough formation. However, if it is negative, troughs may form even with perfectly aligned rollers, when they are oriented in the MD. Figure 6 shows the effect of £$_1$ on the £$_t$ = 10^{-4} contour.

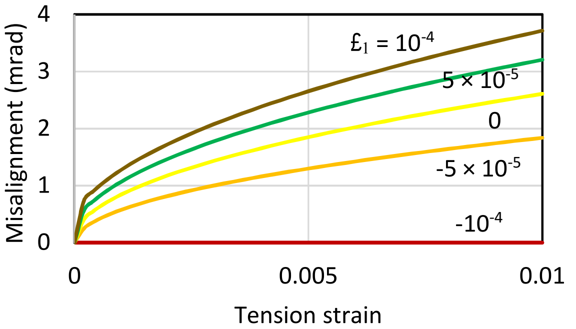

Figure 6: Effect of incoming CD strain £$_1$ on excess width £$_t$ contour of 10^{-4}.

The need for the extra length to exceed a critical value -£$_b$ could be included. The only available estimate is derived from plate buckling theory, as simplified by Shelton [4]. However, in figure 2, the stress acts not in the CD, but perpendicular to the trough. The trough angle increases both the stress along the trough (equation 1) and its length in such a way that those effects cancel, leaving:

$$\sigma_b = E\varepsilon_b = -\frac{\pi h}{L}\sqrt{\frac{E\sigma_x}{3(1-\nu^2)}} \qquad (12)$$

There are very few published data for trough formation, and even these are missing important information to test the theory. The table summarizes the problem for the data on polyester of Beisel [1]:

Effect	**Range**	**Affects**	**Range**
Data from [1]	Using equations 8 and 9	£$_t$	3 – 5 × 10^{-4}
Buckling stress	*W/L* 1-4 (eq. 12) Tension 28-85 N	-£$_b$	1.2 – 8 × 10^{-5}
Roller drag [12]	14 – 140 N.mm	£$_1$	0.6 – 6 × 10^{-5}
Trough visibility	1 - 10 I-units, Slope 0.4 – 1.1°	£$_t$	10^{-5} - 10^{-4}

The data suggest a much larger buckling stress than calculated with equation 12. Roller drag could be significant, but the discrepancy is most likely explained by when troughs are visible in the experiment. Although special viewing conditions were set up, there is no indication given of the threshold. Ideally, trough height would be measured as a function of misalignment, and the data extrapolated back to zero height.

Figure 7 shows trough data for 12 micron thick copper foil from [9]. At these small tension strains, the estimated buckling strain is below 5 × 10^{-7}, and the predicted critical misalignment for trough formation due to buckling (equations 8 and 9) is very low, below 0.025 mrad. If the visibility threshold is 3 × 10^{-4}, the curve of critical angle starts to approach the experimental data. However, the trough angle to achieve this exceeds 28°, the limit of equation 10. This issue recurs below.

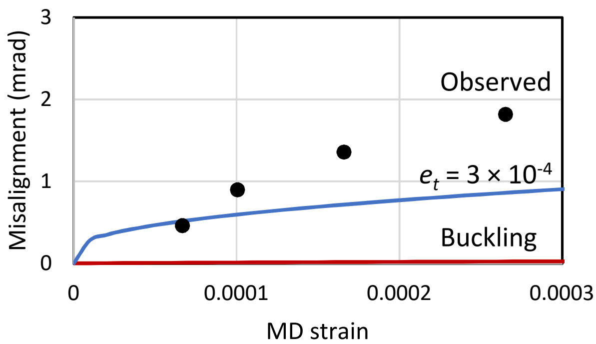

Figure 7: Trough observations in 12 μm copper foil, and predictions for overcoming buckling stress and reaching an excess width of 3 × 10^{-4}.

4. WRINKLES

Wrinkles appear on the downstream roller after they form in the second transition zone around the entry contact line. As above, a simple assumption is that strains do not change through this zone, but the web troughs flatten and the compressive stress increases. If the web on the roller has no wrinkles, its stresses and strains are linked by the usual Hooke's Law. The compressive stress can then be compared with a critical value $\pm_c$ to assess whether a wrinkle will form. With no better theory, the axial stress to buckle a cylindrical shell of radius R [5] will be used, following other authors:

$$\sigma_c = -\frac{Eh}{R}\sqrt{\frac{1}{3(1-\nu^2)}} \quad (13)$$

The compressive stress is directed in the ¸ direction and is given by (ignoring the small contribution of £$_1$):

$$\sigma_\beta = \frac{E}{1-\nu^2}(\varepsilon_\beta + \nu\varepsilon_\alpha) = -\frac{E}{1-\nu^2}\varepsilon_0 t^2(1+t^2) \quad (14)$$

This has similar form to equation 9, so contours of critical stress correspond to the higher £$_t$ values in figure 5.

Model results have been compared with experimental results of Walker et al. [10]. The "wide webs" of their title are not, unfortunately, in the range where equation 11 is valid, with one exception. Figure 8 shows the data on PET and the prediction of the model. The new theory fits the data a little better than the OSU theory implemented in the software package TopWeb [13] reported by Walker. The geometry limit of equation 11 is also plotted, showing that the data are within the range of validity.

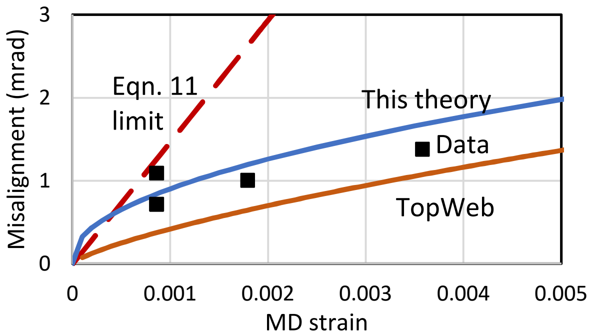

Figure 8: Observation of wrinkles in 12 µm PET, and predictions from this theory and that of Good et al.

The table below shows conditions for both PET in figure 8 and copper foil in figure 9. The results for copper lie above the predictions of this model and TopWeb, which are similar. However, all lie above the geometry limit of equation 11, so the new theory cannot be expected to be accurate.

Material	PET	Copper
Thickness (µm)	12	10
Young's Modulus (GPa)	4.14	105
Span length (m)	1.25	1.25
Web width (m)	1.2	0.4
W/L	0.96	0.32
$\sim_{max}$ – equation 11	25.6°	9.1°

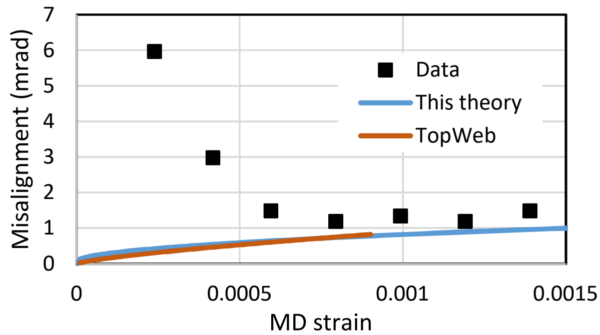

Figure 9: Observation of wrinkles in 10 µm copper foil, and predictions from this theory and that of Good et al.

As MD strain falls, the critical misalignment rises well above the theoretical predictions. This is common to most previous studies [1-3, 9-10]: potential reasons will be discussed later.

5. DISCUSSION

The new theory developed so far relies on there a constant trough angle over a large proportion of the width, favored by a large *W/L* ratio (equation 10). This turned out to be true in only one of the datasets known to the author (figure 8). New experiments on wide, short spans would be informative.

Further development of the theory would include a variation in trough angle, from zero at the edges to a maximum in the center, across a narrow or broad web. This could enable different roller diameter profiles, such as tapered, concave and convex, to be modelled. The troughs remain straight, but may converge or diverge.

The theory predicts steering of a wide web in the troughed state by a misaligned roller, with lateral displacement L^{-}, the same as for a long narrow tape or short, planar span [11]. Perhaps surprisingly, the MD strain is constant throughout the constant trough angle portion, so there is no slackness.

The web enters the downstream roller with shear strain equal to the misalignment angle ¯, and decreased CD width. Both of these provide upstream boundary conditions for the following span. Entering shear strain is equivalent to a roller misalignment [14], and so the second span is likely to be prone to troughs and wrinkles, possibly more than the first. If the wrap around the misaligned roller is 180°, the second span will have the same roller misalignment as the first, so when the shear strain is added it will double the apparent misalignment and make wrinkles more likely.

This propagating shear strain could be removed by returning the web to a planar condition in one long or several shorter spans. This could be achieved by a spreading device, a low friction roller, or an increase in web tension.

A better understanding of web buckling mechanisms would be helpful. The equations borrowed from buckling of a flat panel and free-standing cylindrical shell under axial load seem to work well in many cases, but do not seem totally appropriate to buckling controlled by strain, and buckling at an angle to the CD or roller axis. Quite likely, low amplitude troughs flatten as the web gradually bends in order to wrap the roller. But high amplitude troughs may not flatten before they touch the roller surface, allowing the peaks to push outwards and form a wrinkle.

The predicted shape of the critical misalignment versus tension plots of wrinkle occurrence is very similar to that of the previous theories. However, many experimental curves such as figure 9 and previous experiments [1-3, 9-10] bend upwards at low tension, away from the theory predictions. This is a gradual transition from Regime 1 to Regime 2 behavior, rather than the abrupt termination of the Regime 1 line at a particular tension.

This upward bend was not seen in the PET data of figure 8, so it may require trough angle to be limited by span aspect ratio, i.e. the inequality of equation 11 is not satisfied. Another possibility is that the equations for buckling stress (12) and for the minimum web width to wrinkle need modification for the compressive force acting at large angles to the CD under low tension. A further effect is the gradual loss of friction at lower tension, because of air entrainment. Further progress here depends on either obtaining new experimental data on large *W/L* spans that show this upturn, or developing the theory so it is valid for the existing data sets.

6. CONCLUSIONS

This new theory shows promise for better understanding and prediction of wrinkling, but so far only one set of experimental data falls in its scope of validity. The span must have a large enough *W/L* to produce a central region of constant trough angle. If that is the case, troughs are predicted to exist at misalignment above a critical value which increases with tension (MD) strain. Incoming extra CD strain and Poisson's ratio are the only other factors. There is no dependence on span length, width, thickness or modulus.

Testing the theory for trough occurrence has not been possible, because of the newly-identified need to know the incoming extra strain in the CD (for example, from roller drag), and the threshold amplitude making the troughs visible.

Testing the theory for wrinkle occurrence shows it is reasonable for the one experiment where it is valid. However, to be useful, it needs to be extended to cover smaller *W/L*. Better understanding and models for the buckling into troughs and wrinkles would also be useful.

The effects of a misaligned roller propagate downstream through shear strain, and the following span is likely to add to the wrinkling tendency.

ACKNOWLEDGMENTS

Thanks to Tim Walker of TJW Associates for sharing the files behind the plots in references [9] and [10].

REFERENCES

[1] Beisel, J. A., & Good, J. K., *The Instability of Webs in Transport*. Journal of Applied Mechanics, 78(1), 011001, 2010.

[2] Gehlbach, L. S., Good, J. K., & Kedl, D. M., *Prediction of shear wrinkles in web spans*. TAPPI Journal, 72(8) 1989.

[3] Hashimoto, H., *Theory and Application of Web Handling*, Converting Technical Institute, Tokyo, Japan, 2015.

[4] Shelton J. J., *Buckling of Webs from Lateral Compressive Forces*, Proceedings of the Second International Conference on Web Handling. Ed. Good, J. K., Oklahoma State University, 1993, pp. 303-321.

[5] Timoshenko, S. P. and Gere, J. M., *Theory of Elastic Stability*, McGraw Hill, New York, 1961.

[6] Jones, D. P. and McCann, M. J., *Wrinkling of Webs on Rollers and Drums*. Proceedings of the Eighth International Conference on Web Handling. Ed. Good, J. K., Oklahoma State University, 2005, pp. 123-140.

[7] Fu, B and Good, J. K., *Web Wrinkling Resulting from Moment Transfer*, Proceedings of the 12th International Conference on Web Handling, ed. Good, J. K., Stillwater, OK, USA: Oklahoma State University, 2013.

[8] Wagner, H. (1929). *Flat sheet metal girders with very thin webs*. National Advisory Committee for Aeronautics.

[9] Walker, T. J., Cole, K. A., Zagar, S., & Quass, J. (2011). *Wrinkling of Foils*. In J. K. Good (Ed.), Proceedings of the 11th International Conference on Web Handling (pp. 325-338). Stillwater, OK, USA: Oklahoma State University.

[10] Walker, T. J., & Cole, K. A. (2011). *Wrinkling of Wide Webs*. In J. K. Good (Ed.), Proceedings of the 11th International Conference on Web Handling (pp. 367-385). Stillwater, OK, USA: Oklahoma State University.

[11] Shelton, J. J., *Lateral Dynamics of a Moving Web*, Ph. D. Thesis, Oklahoma State University, Stillwater, OK, USA, 1968.

[12] Walker, T.J., *Practical Application of Idler Roller Performance Measurements and Models*, Proceedings of the Seventh International Conference on Web Handling. Ed. Good, J. K., Oklahoma State University, 2003

[13] Available from Rheologic Ltd, Leeds, UK. www.rheologic.co.uk.

[14] Shelton, J. J., *A Simplified Model for Lateral Behavior of Short Web Spans*, Proceedings of the 6th International Conference on Web Handling, ed. Good, J. K., 2001, pp. 469-484.